Magnetic-field Induced Change of the Viscosity of Diamagnetic Gases with Anisotropic Magnetic Susceptibility

S. Hess

Institut für Theoretische Physik, Universität Erlangen-Nürnberg, Erlangen, Germany

(Z. Naturforsch. **30 a**, 111—116 [1975]; received November 18, 1974)

The influence of a high magnetic field on the viscosity of diamagnetic gases of linear, symmetric and asymmetric top molecules is studied within the frame-work of the kinetic theory based on the Waldmann-Snider equation. The quadratic Zeeman effect associated with the anisotropic magnetic susceptibility is taken into account in addition to the usual linear Zeeman splitting.

A viscous flow in a polyatomic gas gives rise to a collision-induced alignment of the molecular rotational angular momentum. The alignment, in turn, affects the value of the viscosity. Hence application of an external field has an influence on the transport coefficient if the alignment is appreciably altered by the field-induced motion of the rotational angular momentum of a molecule between two successive collisions. This phenomenon, known as the Senftleben-Beenakker effect[1, 2], has been studied extensively both experimentally and theoretically for gases of paramagnetic and diamagnetic linear and symmetric top molecules in the presence of a magnetic field[2], and for polar gases of symmetric top molecules in the presence of an electric field[2]. If the relevant field Hamiltonian is of dipolar type (1st order Zeeman and Stark effects) the field-induced change of the transport coefficients depends on the magnitudes of the magnetic and electric fields H and E and on the pressure of the gas via H/P and E/P. Polar gases of linear $^1\Sigma$-molecules in the presence of an electric field (2nd order Stark effect) where an E^2/P dependence can be expected have been treated theoretically[3] but no measurements have been reported so far.

In this note, theoretical expressions derived from the Waldmann-Snider equation[4] are presented for the magnetic-field-induced change of the viscosity of diamagnetic gases in the presence of strong magnetic fields. The quadratic (2nd order) Zeeman effect associated with the anisotropic molecular susceptibility is taken into account in addition to the usual linear (1st order) Zeeman effect. The quadratic term is of particular importance for heavier (organic) molecules[5]. Linear, symmetric and asymmetric top molecules are treated.

Waldmann-Snider Equation and Field Hamiltonian

The nonequilibrium state of a polyatomic gas is characterized by the distribution function (operator) $f(t, \boldsymbol{x}, \boldsymbol{p}, \boldsymbol{J} \ldots)$ where $\boldsymbol{p}$ and $\hbar \boldsymbol{J}$ are the linear and rotational angular momenta and the dots stand for additional operators specifying the internal state of a molecule. The average $\langle \Psi \rangle$ of an operator $\Psi(\boldsymbol{p}, \boldsymbol{J}, \ldots)$ is defined by $\langle \Psi \rangle = n^{-1} \int \mathrm{d}^3 p \operatorname{Tr} \Psi f$ where $n = \int \mathrm{d}^3 p \operatorname{Tr} f$ is the number density of the gas and "Tr" denotes the trace over all internal quantum numbers. The nonequilibrium distribution f obeys the Waldmann-Snider equation. With $f = f_0(1 + \Phi)$ where f_0 is an equilibrium distribution and Φ characterizes the deviation from equilibrium, the Waldmann-Snider equation can be written as[4, 6, 7] ($\boldsymbol{c}$ is the velocity of a molecule)

$$\frac{\partial \Phi}{\partial t} + \boldsymbol{c} \cdot \nabla \Phi - (i \hbar)^{-1} [\mathcal{H}, \Phi] + \omega(\Phi) = 0 . \quad (1)$$

The 2nd, 3rd and 4th term in Eq. (1) describe the change of Φ due to the free flight, to the internal motion, and to the binary collisions of the molecules. For diamagnetic molecules in the presence of a strong magnetic field $\boldsymbol{H} = H \boldsymbol{h}$ $(\boldsymbol{h} \cdot \boldsymbol{h} = 1)$ the Hamiltonian can be written as[5]

$$\mathcal{H} = -C_1 H \boldsymbol{h} \cdot \boldsymbol{J} - \tfrac{1}{2} C_2 H^2 J^{-1} \boldsymbol{h} \cdot \overline{\boldsymbol{J} \boldsymbol{J}} \cdot \boldsymbol{h} \quad (2)$$

where $\overline{\ldots}$ denotes the symmetric traceless part of a tensor. The coefficients C_1 and C_2 which, in general, depend on the internal quantum numbers of a molecule can be related to the gyromagnetic ratio and the magnetic susceptibility[5], respectively. The commutator $-(i \hbar)^{-1} [\mathcal{H}, \Phi]$ occurring in Eq. (1) can be written as

$$-i \omega_1 [\boldsymbol{h} \cdot \boldsymbol{J}, \Phi] - \frac{i}{2} \omega_2 (\boldsymbol{h} \cdot \hat{\boldsymbol{J}} [\boldsymbol{h} \cdot \boldsymbol{J}, \Phi] + [\boldsymbol{h} \cdot \boldsymbol{J}, \Phi] \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) \quad (3)$$

Reprint requests to Dr. S. Hess, Institut für Theoretische Physik der Universität Erlangen-Nürnberg, *D-8520 Erlangen,* Glückstraße 6.

with $\hat{\boldsymbol{J}} = J^{-1} \boldsymbol{J}$. It is understood that the eigenvalues of J^2 are $j(j+1)$ where j is the rotational quantum number. The eigenvalues of J^{-1} are $[j(j+1)]^{-1/2}$. For gaseous N_2 and gases of heavier molecules at room temperature, the molecules are predominantly in rotational states with $j \geqq 10$. Thus $j(j+1)$ can be approximated by j^2. For large j, (3) can be replaced by

$$-i(\omega_1 + \omega_2 \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) [\boldsymbol{h} \cdot \boldsymbol{J}, \Phi] \,. \tag{3 a}$$

For an asymmetric top molecule the frequencies ω_1 and ω_2 are given by

$$\hbar \omega_1 = \mu_N g H \,, \qquad \hbar \omega_2 = \chi' H^2 \tag{4}$$

with

$$g = \sum_{i=1}^{3} \alpha_i g_i \,, \qquad \chi' = \tfrac{3}{2} J^{-1} \sum_{i=1}^{3} \alpha_i (\chi_i - \chi) \,, \tag{5}$$

$$\alpha_i = J^{-2} (\boldsymbol{e}_i \cdot \boldsymbol{J})^2 = (\boldsymbol{e}_i \cdot \hat{\boldsymbol{J}})^2 \,. \tag{6}$$

The $\boldsymbol{e}_i$ are unit vectors parallel to the principal axes of the moment of inertia tensor. The g_i and χ_i are the diagonal elements of the molecular g-value tensor and of the susceptibility tensor in the principal inertial axes system. The quantity χ is given by $\chi = \frac{1}{3} \Sigma \chi_i$. The nuclear magneton has been denoted by μ_N. For a symmetric top molecule one has $g_1 = g_2 = g_\perp$, $g_3 = g_\parallel$; $\chi_1 = \chi_2 = \chi_\perp$, $\chi_3 = \chi_\parallel$ where parallel and perpendicular refers to the molecular figure axis. In this case Eq. (5) reduces to

$$g = (1 - \alpha_3) g_\perp + \alpha_3 g_\parallel \,; \qquad \chi' = \tfrac{1}{2} J^{-1} (1 - 3\alpha_3)(\chi_\perp - \chi_\parallel) \,. \tag{7}$$

Here, the eigenvalues of α_3 are $[j(j+1)]^{-1} k^2$, $-j \leqq k \leqq j$. The expressions for a linear molecule are obtained from (7) with $\alpha_3 = 0$.

The precession frequency of the rotational angular momentum associated with the 1st order Zeeman effect is ω_1. The quantity $\omega_1 + \omega_2 \boldsymbol{h} \cdot \hat{\boldsymbol{J}}$ can be looked upon as a precession frequency which depends on the angle between $\boldsymbol{J}$ and the applied magnetic field.

Calculation of the Viscosity Coefficients

For all gases measured so far (except NH_3) the dominant type of alignment set up by a viscous flow and the only one considered here is characterized by the (2nd rank) tensor polarization (alignment)

$$\mathbf{a} = \langle \boldsymbol{\Phi} \rangle \,, \ \boldsymbol{\Phi} = \sqrt{\tfrac{15}{2}} \, [J^2 (J^2 - 3/4)]^{-1/2} \, \overline{\boldsymbol{J}\boldsymbol{J}} \,. \tag{8}$$

For convenience, the normalization factor occurring in (8) has been chosen such that $\boldsymbol{\Phi} : \boldsymbol{\Phi} = 5$. Transportrelaxation equations for the friction pressure tensor $\overline{\mathbf{p}}$ and the tensor polarization can be derived from the Waldmann-Snider equation by application of the moment method [8, 9]. The quadratic Zeeman term in the Hamiltonian (2) couples $\mathbf{a}$ with the vector polarization $\langle J^{-1} \boldsymbol{J} \rangle$ and the 3rd rank tensor polarization $\langle J^{-3} \overline{\boldsymbol{J}\boldsymbol{J}\boldsymbol{J}} \rangle$. Thus similar complications as discussed in Ref. [3] (see also Ref. [10]) in connection with the 2nd order Stark effect arise here. If the relaxation frequency of the n-th rank tensor polarization $(n = 1, 3, 4, \ldots)$ is approximately equal to the relaxation frequency ω_a of the 2nd rank tensor polarization a simpler solution procedure can be used. This method works as follows.

Firstly, the usual transport-relaxation equation is derived for the friction pressure tensor $\overline{\mathbf{p}} = n \langle \overline{\boldsymbol{c}\boldsymbol{p}} \rangle$ ($\boldsymbol{p} = m\,\mathbf{c}$, m is the mass of a molecule). To this purpose, Eq. (1) is multiplied by $f_0 \overline{\boldsymbol{c}\boldsymbol{p}}$ and then the integration over the linear momentum and the trace "Tr" is performed. Subject to the assumption that the tensor polarization $\mathbf{a}$ is the only type of collision-induced alignment which is produced by $\overline{\mathbf{p}} \neq 0$, and if spatial gradients of nonconserved quantities are disregarded, one obtains [8, 9]

$$\frac{\partial \overline{\mathbf{p}}}{\partial t} + 2P \overline{\nabla \boldsymbol{v}} + \omega_p \overline{\mathbf{p}} + \omega_{pa} \sqrt{2} P \mathbf{a} = 0 \,. \tag{9}$$

Here $P = n k_B T$ and $\boldsymbol{v} = \langle \mathbf{c} \rangle$ are the equilibrium pressure and the flow velocity of the gas; ω_p and ω_{pa} are relaxation coefficients which can be expressed in terms of collision integrals evaluated with the Waldmann-Snider collision operator. Some of their properties will be discussed later.

Next, an equation for $\mathbf{a}$ is needed. To circumvent the above mentioned difficulties connected with the 2nd order Zeeman term, an equation for $\mathbf{a} : \boldsymbol{\Phi}$ [cf. Eq. (8)] is first derived from Eq. (1) rather than the usual transport-relaxation equation [8, 9] for $\mathbf{a}$. To this purpose, Eq. (1) is multiplied by f_0 and integrated over the linear momentum. Subject to the same assumption which led to Eq. (9) one finds

$$\boldsymbol{\Phi} : \partial \mathbf{a} / \partial t - i(\omega_1 + \omega_2 \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) \, [\boldsymbol{h} \cdot \boldsymbol{J}, \boldsymbol{\Phi}] : \mathbf{a} + \boldsymbol{\Phi} : \left[\frac{\omega_{ap}}{\sqrt{2} P} \overline{\mathbf{p}} + \omega_a \mathbf{a} \right] = 0 \,, \tag{10}$$

with two additional relaxation coefficients ω_a and ω_{ap}. Time reversal invariance of the molecular interaction implies the Onsager relation $\omega_{ap} = \omega_{pa}$.

The inequalities $\omega_p > 0$, $\omega_a > 0$, and $\omega_p \, \omega_a > \omega_{ap}{}^2$ guarantee the decay to equilibrium.

Equation (10) can be cast into a more tractable form with the help of the 4-th rank projection tensors [9] $\mathcal{P}^{(m)}$ $(m = 0, \pm 1, \pm 2)$. They describe the rotation of a 2nd rank tensor about a direction specified by the unit vector $\boldsymbol{h}$. The definitions

$$\mathbf{a}^{(m)} = \mathcal{P}^{(m)} : \mathbf{a} \, , \quad \boldsymbol{\Phi}^{(m)} = \mathcal{P}^{(m)} : \boldsymbol{\Phi} \tag{11}$$

are introduced. Furthermore, use is made of the following properties

$$\mathcal{P}^{(m)} : \mathcal{P}^{(m')} = \delta_{mm'} \, \mathcal{P}^{(m)} \, , \quad \sum \mathcal{P}^{(m)} = 1 \, , \sum \mathbf{a}^{(m)} = \mathbf{a} \, , \tag{12}$$

$$\boldsymbol{\Phi} : \mathbf{a}^{(m)} : = \boldsymbol{\Phi}^{(-m)} : \mathbf{a}^{(m)} \, , \tag{13}$$

$$[\boldsymbol{h} \cdot \boldsymbol{J}, \, \boldsymbol{\Phi}^{(m)}] = i \, m \, \boldsymbol{\Phi}^{(m)} \, . \tag{14}$$

Thus, Eq. (10) is equivalent to

$$\begin{aligned} &\boldsymbol{\Phi}^{(-m)} : \partial \mathbf{a}^{(m)} / \partial t \\ &\quad + [i \, m (\omega_1 + \omega_2 \, \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) + \omega_a] \, \boldsymbol{\Phi}^{(-m)} : \mathbf{a}^{(m)} \\ &\quad + \frac{\omega_{ap}}{\sqrt{2} \, P} \, \boldsymbol{\Phi}^{(-m)} : \overline{\mathbf{p}}^{(m)} = 0 \, , \end{aligned} \tag{15}$$

where $\overline{\mathbf{p}}^{(m)}$ is defined analogous to (11).

For the treatment of the viscosity problem, a steady state situation is considered, i.e. the time derivatives in Eqs. (9), (10), (15) are put equal to zero. Then Eq. (15) yields

$$\begin{aligned} \boldsymbol{\Phi}^{(-m)} : \mathbf{a}^{(m)} = {} & [1 + i \, m \, (\varphi_1 + \varphi_2 \, \boldsymbol{h} \cdot \hat{\boldsymbol{J}})]^{-1} \\ & \cdot \frac{\omega_{ap}}{\sqrt{2} \, P \, \omega_a} \, \boldsymbol{\Phi}^{(-m)} : \overline{\mathbf{p}}^{(m)} \, , \end{aligned} \tag{16}$$

with the abbreviations

$$\varphi_{1,2} = \omega_{1,2} / \omega_a \, . \tag{17}$$

The alignment tensor $\mathbf{a}^{(m)}$ needed in Eq. (9) is now obtained from Eq. (16). This equation is multiplied by $Q^{-1} \exp\{-H_{\text{rot}}/k_B \, T\} \; \boldsymbol{\Phi}$ and the trace "Tr" is performed. Here H_{rot} is the rotational Hamiltonian and Q is given by $Q = \text{Tr} \exp\{-H_{\text{rot}}/k_B \, T\}$.

The resulting equation is

$$\mathbf{a}^{(m)} = L_m(\varphi_1, \varphi_2) \, \frac{\omega_{ap}}{\sqrt{2} \, P \, \omega_a} \, \overline{\mathbf{p}}^{(m)} \tag{18}$$

with

$$\begin{aligned} & L_m(\varphi_1, \varphi_2) \\ & \quad = \langle [1 + i \, m (\varphi_1 + \varphi_2 \, \boldsymbol{h} \cdot \hat{\boldsymbol{J}})]^{-1} \, \boldsymbol{\Phi}^{(m)} : \boldsymbol{\Phi}^{(-m)} \rangle_0 \end{aligned} \tag{19}$$

where $\langle \ldots \rangle_0$ indicates an equilibrium average. Notice that $L_m(0, 0) = 1$. Insertion of (18) into Eq. (9), finally yields, for a steady state situation

$$\overline{\mathbf{p}} = -2 \sum_{m=-2}^{2} \eta^{(m)} \, \mathcal{P}^{(m)} : \overline{\nabla \boldsymbol{v}} \, , \tag{20}$$

with the complex viscosity coefficients

$$\eta^{(m)} = \eta^{(m)}(\varphi_1, \varphi_2) = \frac{P}{\omega_p} \, [1 + A \, L_m(\varphi_1, \varphi_2)] \, . \tag{21}$$

The positive quantity A is essentially a measure for the strength of the collision-induced coupling between $\overline{\mathbf{p}}$ and $\mathbf{a}$. It is defined by

$$A = \omega_{pa} \, \omega_{ap} / \omega_p \, \omega_a \, . \tag{22}$$

In Eq. (21) terms nonlinear in A have been disregarded since A can be expected to be of the order 10^{-2} or even smaller. The possibilities to measure the 5 independent viscosity coefficients have been discussed in Refs. [9] and [11].

Magnetic Field and Pressure Dependence of the Viscosity

For the Senftleben-Beenakker effect, the relative magnetic-field induced change of the viscosity, i.e.

$$\varepsilon_m = \eta^{-1} \, [\eta^{(m)} - \eta] \tag{23}$$

is of interest where $\eta = (P/\omega_p)(1 + A)$ is the field free viscosity. From Eqs. (19), (21) one infers

$$\text{Re} \, \varepsilon_m = -A [1 - \langle f(m \, \varphi_1 + m \, \varphi_2 \, \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) \, K_m \rangle_0] \, , \tag{24}$$

$$\text{Im} \, \varepsilon_m = -A \, \langle g(m \, \varphi_1 + m \, \varphi_2 \, \boldsymbol{h} \cdot \hat{\boldsymbol{J}}) \, K_m \rangle_0 \, , \tag{25}$$

$$\text{with} \quad f(x) = (1 + x^2)^{-1} \, , \; g(x) = x \, f(x) \, , \tag{26}$$

$$K_m = \boldsymbol{\Phi}^{(m)} : \boldsymbol{\Phi}^{(-m)} \, . \tag{27}$$

Notice that $\varepsilon_0 = 0$. In Eqs. (24), (25), averages over the magnetic, the rotational and the k-quantum numbers have to be performed. In this connection it is of importance to note that

$$K_1 = K_{-1} = \tfrac{15}{2} \, [(\boldsymbol{h} \cdot \hat{\boldsymbol{J}})^2 - (\boldsymbol{h} \cdot \hat{\boldsymbol{J}})^4] \, , \tag{28}$$

and

$$K_2 = K_{-2} = \tfrac{15}{8} \, [1 - (\boldsymbol{h} \cdot \hat{\boldsymbol{J}})^2]^2 \, . \tag{29}$$

Expressions similar to (24), (25) with (28), (30) have been obtained previously [12] for the Senftleben-effect [13] of the paramagnetic gas O_2 subject to approximations referred to as the "uncoupled model".

Next, the dependence of ε_m on the magnitude H of the magnetic field and on the number density n is considered. The relaxation coefficients $\omega_p, \omega_a, \ldots$ are proportional to the number density. Hence φ_1

and φ_2 are proportional to H/n and H^2/n, respectively. At constant temperature, one has $\varphi_1 \sim H/P$ and $\varphi_2 \sim H^2/P$ where P is the pressure. Thus the presence of the 2nd order Zeeman effect leads to deviations from the usual H/P dependence of the Senftleben-Beenakker effect.

The relative importance of the 2nd order Zeeman effect for the magnetic field dependence of the viscosity is determined by the ratio $\varphi_2^2/\varphi_1^2 = \omega_2^2/\omega_1^2$. To obtain an appreciation for the orders of magnitude involved the ratio of the averaged quantities

$$\omega_{10}^2 = \langle \omega_1^2 \rangle_0 , \quad \omega_{20}^2 = \langle \omega_2^2 \rangle_0 \tag{30}$$

and

$$\varphi = \omega_{10}/\omega_a , \quad \psi = \omega_{20}/\omega_a \tag{31}$$

is considered. With

$$g_0^2 = \langle g^2 \rangle_0 , \; (\Delta\chi)^2 = j_0^2 \langle \chi'^2 \rangle_0 , \; j_0^2 = \langle J^2 \rangle_0 , \tag{32}$$

one has

$$\psi^2/\varphi^2 = \omega_{20}^2/\omega_{10}^2 = (\Delta\chi \, H/j_0 \, g_0 \, \mu_N)^2 . \tag{33}$$

A crude estimate is made for the gases N_2 and C_6H_5F (fluorobenzene) where the values $g_0 \approx 0.26$, 0.034; $\Delta\chi \approx 7 \times 10^{-30}$, 80×10^{-30} erg G^{-2}, and $j_0 \approx 12, 70$ (room temperature) apply. For a magnetic field of 100 kG the precession frequencies associated with the 1st and 2nd order Zeeman effects are $\omega_{10} \approx 130 \times 10^6$, $17 \times 10^6 \, s^{-1}$, and $\omega_{20} \approx 6 \times 10^6$, $12 \times 10^6 \, s^{-1}$.

The resulting ratio ψ/φ is equal to 0.05 and 0.7 for N_2 and C_6H_5F, respectively. Hence the quadratic Zeeman effect can be disregarded for N_2 even at 100 kG. This, however, is not the case for a gas of molecules like C_6H_5F.

Approximate Explicit Expressions for ε_m

In general, the averages over rotational and k-quantum numbers occurring in Eqs. (24), (25) have to be calculated numerically for given ω_1 and ω_2. For the case where $\varphi_2^2 \approx \psi^2$ holds true, simpler expressions for ε_m can be obtained if $\langle \varphi_1^{2n} \rangle_0$ and $\langle \varphi_1^{2n+1} \rangle_0$ are replaced by φ^{2n} and $\langle \varphi_1 \rangle_0 \, \varphi^{2n}$. These relations are exact for linear molecules where one has $\varphi_1 = \langle \varphi_1 \rangle_0 = \varphi$. This approximation should reveal the essential features of the quadratic Zeeman effect on the magnetic field dependence of the transport coefficients for molecules which are almost linear and for molecules which have a small spread in g_i-values. Furthermore, it is assumed that j_0 is large enough such that the summation over the magnetic quantum numbers can be replaced by an integration. The resulting approximate expressions for ε_m are

$$\operatorname{Re} \varepsilon_m \approx - A \, F_m(\varphi, \psi) , \tag{34}$$

$$\operatorname{Im} \varepsilon_m \approx - A \, \frac{\langle \varphi_1 \rangle_0}{\varphi} \, G_m(\varphi, \psi) , \tag{35}$$

with

$$F_m(\varphi, \psi) = 1 - \tfrac{1}{2} \int_{-1}^{+1} \mathrm{d}x \, f(m\varphi + m\psi x) K_m , \tag{36}$$

$$G_m(\varphi, \psi) = \tfrac{1}{2} \int_{-1}^{+1} \mathrm{d}x \, g(m\varphi + m\psi x) K_m . \tag{37}$$

In Eqs. (36), (37), it is understood that $\boldsymbol{h} \cdot \hat{\boldsymbol{J}}$ occurring in K_m (cf. 28, 39) is replaced by the continous variable x $(-1 \leqq x \leqq 1)$. The integrals (36), (37) can be evaluated analytically. The results are

$$F_1 + i\,G_1 = 1 - \frac{15}{2} \frac{(1+i\varphi)^2}{\psi^4} \left\{ 1 + i\varphi + \frac{2}{3} \frac{\psi^2}{1+i\varphi} - [(1+i\varphi)^2 + \psi^2] \, l(\varphi, \psi) \right\} , \tag{38}$$

$$\begin{aligned} F_2 + i\,G_2 = 1 - \frac{15}{8} \frac{(1+2i\varphi)^2}{16\,\psi^4} \Bigg\{ & 1 + 2i\varphi + \frac{5}{3} \frac{4\,\psi^2}{1+2i\varphi} (1+2i\varphi)^2 \\ & - \left[(1+2i\varphi)^2 + 8\,\psi^2 + \frac{16\,\psi^4}{(1+2i\varphi)^2} \right] l(2\varphi, 2\psi) \Bigg\} , \end{aligned} \tag{39}$$

with

$$l(x, y) = \frac{1}{2iy} \ln \frac{1+ix+iy}{1+ix-iy} = \frac{1}{y} \arctan \frac{y}{1+ix} . \tag{40}$$

For $\psi = 0$, (38) and (39) reduce to $1 - (1+i\varphi)^{-1}$ and $1 - (1+2i\varphi)^{-1}$.

For the purpose of plotting F_m and G_m, a numerical integration of (36), (37) is more convenient. Next, it is noticed that ψ can either be written as

$$\psi = (n/n^*) \, \varphi^2 \tag{41}$$

or as (cf. 33)

$$\psi = (H/H^*) \, \varphi . \tag{42}$$

The reference number density n^* and the reference magnetic field strength H^* are given by

$$n^* = \mu_N^2 \, g_0^2 \, j_0^2 \, (v_{th} \, \sigma_a \, \hbar \, \Delta\chi)^{-1} , \tag{43}$$

$$H^* = \mu_N \, g_0 \, j_0 \, (\Delta\chi)^{-1} . \tag{44}$$

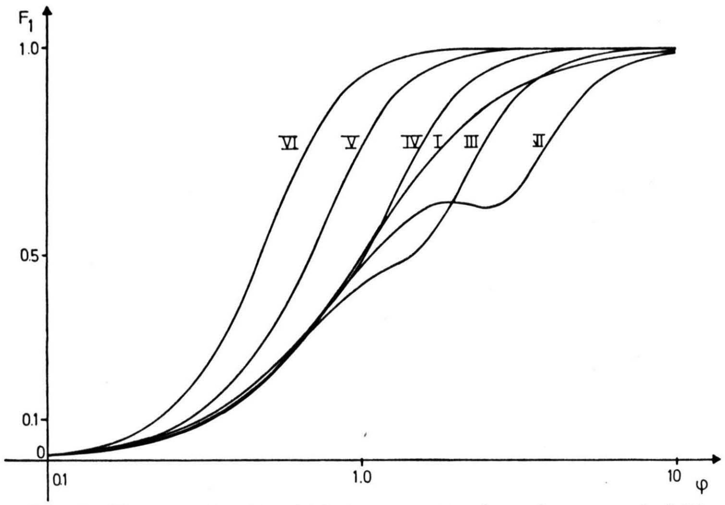

Fig. 1. The quantity F_1 which is proportional to the magnetic-field-induced change of the viscosity coefficient Re η_1 versus $\varphi \sim H/n$. The curves I—VI pertain to the values 0, 0.5, 1, 2, 4, 8 for the parameter $\beta = n/n^*$ which characterizes the relative importance of the 2nd order Zeeman effect.

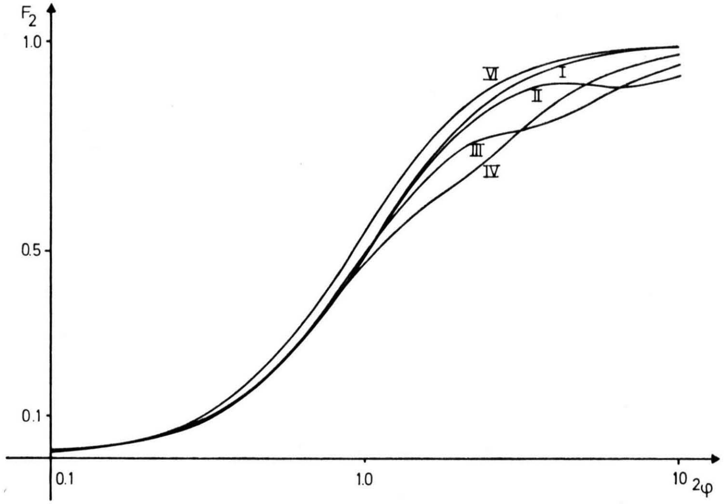

Fig. 3. The quantity F_2 which is proportional to the magnetic-field-induced change of the viscosity coefficient Re η_2 versus $2\,\varphi$. Notice that curve V has been omitted.

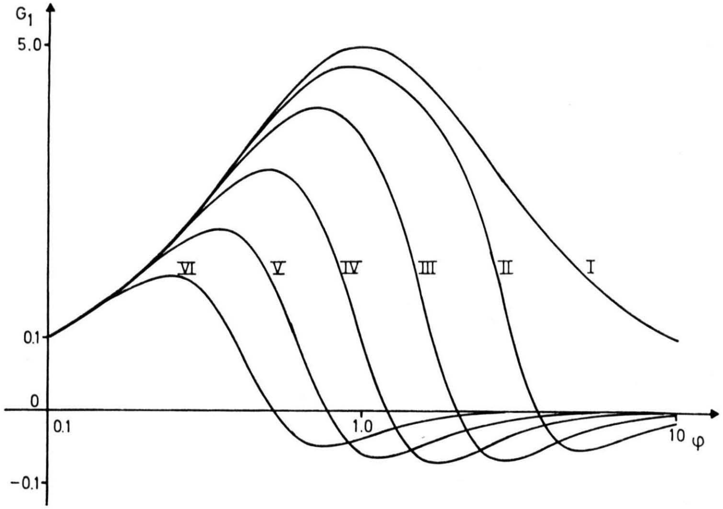

Fig. 2. The quantity G_1 determining the transverse viscosity coefficient Im η_1 versus φ. The values for the parameter β are the same as in Figure 1.

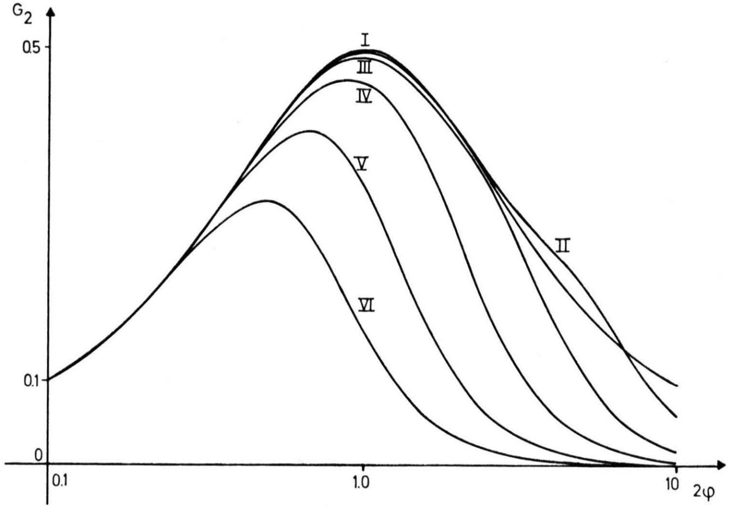

Fig. 4. The quantity $G_2 \sim$ Im η_2 versus $2\,\varphi$. The parameter β has the same values as in Figure 1.

In (43), v_{th} is a thermal velocity. The effective cross section σ_a is defined by $\omega_a = n\, v_{th}\, \sigma_a$. Relations (41), (42) indicate that the 2nd order Zeeman effect gives rise to an extra density dependence or an extra H dependence in addition to the usual H/n or H/P dependence of the viscosity unless one has $n \ll n^*$ or $H \ll H^*$. For N_2 and C_6H_5F the characteristic magnetic field strength H^* is of the order of 2000 kG and of 140 kG, respectively. A crude estimate for n^*, as given by (43), yields, with $\sigma_a \approx 50 \times 10^{-16}$ and $150 \times 10^{-16}\,\text{cm}^2$, $n^* \approx 10^{19}\,\text{cm}^{-3}$ and $3 \times 10^{16}\,\text{cm}^{-3}$ for N_2 and C_6H_5F, respectively. These number densities correspond to pressures of roughly 300 Torr and 20 Torr at room temperature.

In Figs. 1 – 4, F_m and G_m $(m = 1, 2)$ as given by (36), (37) have been plotted as functions of $m\varphi$. The parameter is the ratio $\beta = n/n^*$, cf. Equation (41). The curves labelled by I to VI pertain to $\beta = 0$, 0.5, 1, 2, 4, 8, respectively. The value $\beta = 0$ (curves I) corresponds to the case where the 2nd order Zeeman effect is absent. Notice that the influence of the 2nd order Zeeman effect is more pronounced for F_1 and G_1 than for F_2 and G_2. Furthermore, it is obvious that G_1 and G_2 which are associated with the transverse viscosity coefficients are more dramatically affected by the 2nd order Zeeman effect than F_1 and F_2.

Final remarks

The influence of a magnetic field on the heat conductivity of a diamagnetic gas can be treated theoretically similar to the magnetic-field-induced change of the viscosity which has been considered in this note. Measurements of the influence of strong magnetic fields on the transport properties are desirable for gases of molecules where the 2nd order Zeeman effect associated with the anisotropic magnetic susceptibility is expected to play an important role. This is the case e. g. for C_6H_5F, C_6H_6 and heavier organic molecules in fields over 100 kG.

Finally, it seems worth mentioning that the expressions (24), (25) and (34) – (39) can also be applied to polar gases of linear $^1\Sigma$-molecules in the presence of an electric field (2nd order Stark effect), as well as to the case of a spatially inhomogeneous electric field (where the molecular quadrupole energy has to be taken into account) if ω_2 is appropriately modified. In these case no transverse effect exists ($\eta^{(m)}$ real) unless a magnetic field is present at the same time.

Acknowledgements

Many thanks are due to N. Müller for programming the computer-drawn curves of Figures 1 – 4. Furthermore, I would like to thank Dr. W. Hüttner, University of Ulm, for a conversation where he pointed out to me that the 2nd order Zeeman effect is of much greater importance for molecules like C_6H_5F and C_6H_6 than for molecules like H_2 and N_2.

---

[1] H. Senftleben, Phys. Z. **31**, 922, 961 [1930]; J. J. M. Beenakker e. a. Phys. Lett. **2**, 5 [1962].

[2] For reviews see J. J. M. Beenakker and F. R. McCourt, Ann. Rev. Phys. Chem. **21**, 47 [1970]; J. J. M. Beenakker, in The Boltzmann Equation-Theory and Application, eds. E. G. D. Cohen and W. Thirring, Springer, Wien-New York 1973.

[3] S. Hess and F. R. McCourt, Physica **44**, 19 [1969].

[4] L. Waldmann, Z. Naturforsch. **12 a**, 660 [1957]; **13 a**, 609 [1958]. — R. F. Snider, J. Chem. Phys. **32**, 1051 [1960].

[5] A review of molecular *g*-values and magnetic susceptibilities for over 100 molecules has been given by W. H. Flygare and R. C. Benson, Mol. Phys. **20**, 225 [1971]. For the theory of the linear and quadratic Zeeman effects in asymmetric top molecules see W. Hüttner and W. H. Flygare, J. Chem. Phys. **47**, 4137 [1967].

[6] S. Hess, Z. Naturforsch. **22 a**, 1871 [1967].

[7] S. Hess, Z. Naturforsch. **25 a**, 350 [1970]; Springer Tracts Mod. Phys. **54**, 136 [1970].

[8] S. Hess and L. Waldmann, Z. Naturforsch. **21 a**, 1529 [1966]. — H. Raum and W. E. Köhler, Z. Naturforsch. **25 a**, 1178 [1970].

[9] S. Hess and L. Waldmann, Z. Naturforsch. **25 a**, 1367 [1970]; **26 a**, 1057 [1971].

[10] F. Tommasini, A. C. Levi, e. a. Physica **49**, 299 [1970].

[11] H. Hulsman and H. F. P. Knaap, Physica **50**, 565 [1970].

[12] J. J. M. Beenakker, J. A. R. Coope, and R. F. Snider, Phys. Rev. A, **4**, 788 [1971].

[13] H. Hulsman, A. L. J. Burgmans, E. J. van Waasdijk, and H. F. Knaap, Physica **50**, 558 [1970].